Parameterized Meshes

Lecture 22

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Outline

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Surfaces Defined by Functions

- Let $F(x, y, z) = 0$ be an equation that implicitly defines a 2-dimensional surface in 3-dimensional space.
- For example,
  - $x^2 + y^2 + z^2 - 1 = 0$ defines a sphere.
  - $x^2 + z^2 - y^2 = 0$ defines a cone.

Parameterizing the Surface

- Let x, y, and z be given in terms of parameters s and t, with $a \leq s \leq b$, $c \leq t \leq d$.
  - $x = x(s, t)$
  - $y = y(s, t)$
  - $z = z(s, t)$
- Typically, $[a, b]$ and $[c, d]$ are $[0, 1]$ or $[0, 2\pi]$.
- Then $P(s, t) = (x, y, z)$ is a point on the surface.

Outline

A Sphere of Radius 1

Example (A Sphere of Radius 1)

- Use spherical coordinates, where
  - s is the angle around the vertical axis (longitude),
  - t is the angle up from the equator (latitude).
- Then let

$$x = \cos t \sin s$$
$$y = \sin t$$
$$z = \cos t \cos s,$$

with $0 \leq s \leq 2\pi$ and $-\frac{\pi}{2} \leq t \leq \frac{\pi}{2}$.

A Sphere of Radius 1

Example (A Sphere of Radius 1)

- Then

$$P(s,t) = (\cos t \sin s, \sin t, \cos t \cos s)$$

  is a point on the surface.

- Check:

$$\begin{aligned} x^2 + y^2 + z^2 - 1 &= (\cos t \sin s)^2 + (\sin t)^2 + (\cos t \cos s)^2 - 1 \\ &= \cos^2 t \sin^2 s + \sin^2 t + \cos^2 t \cos^2 s - 1 \\ &= \cos^2 t \left(\sin^2 s + \cos^2 s\right) + \sin^2 t \\ &= \cos^2 t + \sin^2 t - 1 \\ &= 1 - 1 \\ &= 0. \end{aligned}$$

Outline

A Paraboloid

Example (A Paraboloid)

- s represents an angle around the central axis.
- t represents the radius of a circular horizontal cross-section.
- The paraboloid of height 1 is defined by

$$x = t \sin s$$
$$y = t^2$$
$$z = t \cos s$$

with $0 \leq s \leq 2\pi$ and $0 \leq t \leq 1$.

A Paraboloid

Example (A Paraboloid)

- Then

$$P(s,t) = (t\sin s, t^2, t\cos s)$$

  is a point on the surface of the paraboloid.

- Check:

$$\begin{aligned} x^2 + z^2 - y &= t^2\sin^2 s + t^2\cos^2 s - t^2 \\ &= t^2\left(\sin^2 s + \cos^2 s\right) - t^2 \\ &= t^2 - t^2 \\ &= 0. \end{aligned}$$

Outline

Assignment

Assignment

- Assignment 18.